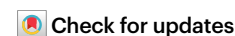


Designing flexible mechanical metamaterials with complex functionalities

Gary P. T. Choi



A technique has been developed for automatically discovering mechanical metamaterials with desired nonlinear dynamic responses.

Mechanical metamaterials are artificially engineered materials with highly unusual physical and mechanical properties. In recent years, the design of mechanical metamaterials has been extensively studied^{1,2} for achieving various advanced functionalities such as topological protection³, cloaking⁴ and focusing⁵. However, most previous design approaches have focused on only a single functionality and operate in the linear regime. Now, writing in *Nature Materials*, Giovanni Bordiga and colleagues⁶ propose an inverse design framework for creating flexible mechanical metamaterials with different target nonlinear dynamic responses and reconfigurable functionalities.

The proposed inverse design framework considers rigid units connected by flexible ligaments in a fixed topology, such as a grid-like topology or a kagome-like topology (Fig. 1a). It then uses the shape of the rigid units as the design space to search for a metamaterial geometry with the desired nonlinear dynamic responses. Specifically, the design space is parameterized by the coordinates of the vertices of the units in the reference configuration. Constraints on the edge lengths, the vertex angles of the rigid units and the angles between neighbouring units are further imposed to avoid infeasible designs. Then, one can set different objective functions based on the desired effects and solve the constrained optimization problem to obtain the optimized geometry. Here, unlike most previous gradient-based⁷ and gradient-free⁸ methods for material design, the authors utilize automatic differentiation^{9,10} for the optimization.

The main benefit of the inverse design framework reported by Bordiga and colleagues is the wide range of nonlinear dynamic tasks that the metamaterial design can achieve. For instance, it is possible to optimize the metamaterial design to direct the kinetic energy provided by an input pulse towards a target region, thereby achieving energy focusing (Fig. 1b). Specifically, this can be done by maximizing the time integral of the kinetic energy at the target region over the coordinates of all vertices. Conversely, it is also possible to achieve energy splitting using the proposed inverse design framework (Fig. 1c), where the metamaterial geometry is optimized such that the energy provided by a dynamic boundary excitation is split between two target regions. The key is to formulate an objective function $J = w_1 J_1 + w_2 J_2$, where J_1 and J_2 are the time integrals of the kinetic energy at the two target regions respectively, and w_1 and w_2 are the weighting parameters with $w_1 \in [0, 1]$ and $w_2 = 1 - w_1$, and then perform the constrained optimization.

In many real-world problems, it is desirable for metamaterial structures to show different behaviours under a simple change of

certain conditions. Here, using the high deformability of mechanical metamaterials, it is possible to achieve such tunability in an optimized metamaterial design. Specifically, one can reprogramme the energy-focusing location under different levels of applied pre-compression (Fig. 1d) using the proposed inverse design framework. This is done by formulating an objective function $J = w_1 \int_{\Omega_{t1}}^{\epsilon_1} + w_2 \int_{\Omega_{t2}}^{\epsilon_2}$, where w_1 and w_2 are the weighting parameters, Ω_{t1} and Ω_{t2} are the two target regions, and $\int_{\Omega_{t1}}^{\epsilon_1}$ and $\int_{\Omega_{t2}}^{\epsilon_2}$ are the time integrals of the kinetic energy at the two target regions with the structure pre-compressed at two different pre-compression levels, ϵ_1 and ϵ_2 , respectively. Another possible multi-task effect that can be achieved by the inverse design framework is multi-input energy focusing (Fig. 1e). The optimized metamaterial architecture is capable of focusing energy at a given target region when excited at multiple independent boundary locations. The optimal design is identified by maximizing the sum of all time integrals of the kinetic energy at the target region corresponding to the different boundary excitations.

Using the inverse design framework, one can also reprogramme the functionality of the designed metamaterials to perform antagonistic tasks such as focusing/protection switching (Fig. 1f). Here the metamaterial architecture is optimized to maximize kinetic energy at a target region under an applied pre-compression, while simultaneously minimizing the energy at the same region under a different level of applied pre-compression. This is done by maximizing an objective function $J = w_1 \int_{\Omega_t}^{\epsilon_1} + w_2 \int_{\Omega_t}^{\epsilon_2}$, where Ω_t is the target region, ϵ_1 is the level of applied pre-compression for which the kinetic energy is desired to be maximized and ϵ_2 is the other level of pre-compression for which the kinetic energy is desired to be minimized. The weighting parameters are set to be $w_1 \in [0, 1]$ and $w_2 = w_1 - 1 \in [-1, 0]$ so that maximizing J would achieve the two antagonistic tasks of energy maximization and minimization simultaneously.

While the above-mentioned tasks are primarily about energy-focusing and some related controls, it is also possible to utilize the proposed inverse design framework for other tasks such as nonlinear motion conversion (Fig. 1g), in which the metamaterial architecture is optimized to transform a time-harmonic linearly polarized input excitation into a circularly polarized motion at a target region. The key is to formulate an objective function containing the angular momentum of the rigid units with respect to the midpoint of the target region. Then, by maximizing (or minimizing) the objective function, one can obtain a desired anticlockwise (or clockwise) circularly polarized motion at the target region.

All together, Bordiga and colleagues have developed an efficient computational framework for designing reprogrammable nonlinear dynamic metamaterials, thereby demonstrating the capability of mechanical metamaterials for a broad spectrum of nonlinear dynamic tasks and functionalities. It would be of great interest to further investigate the theoretical properties of the proposed design

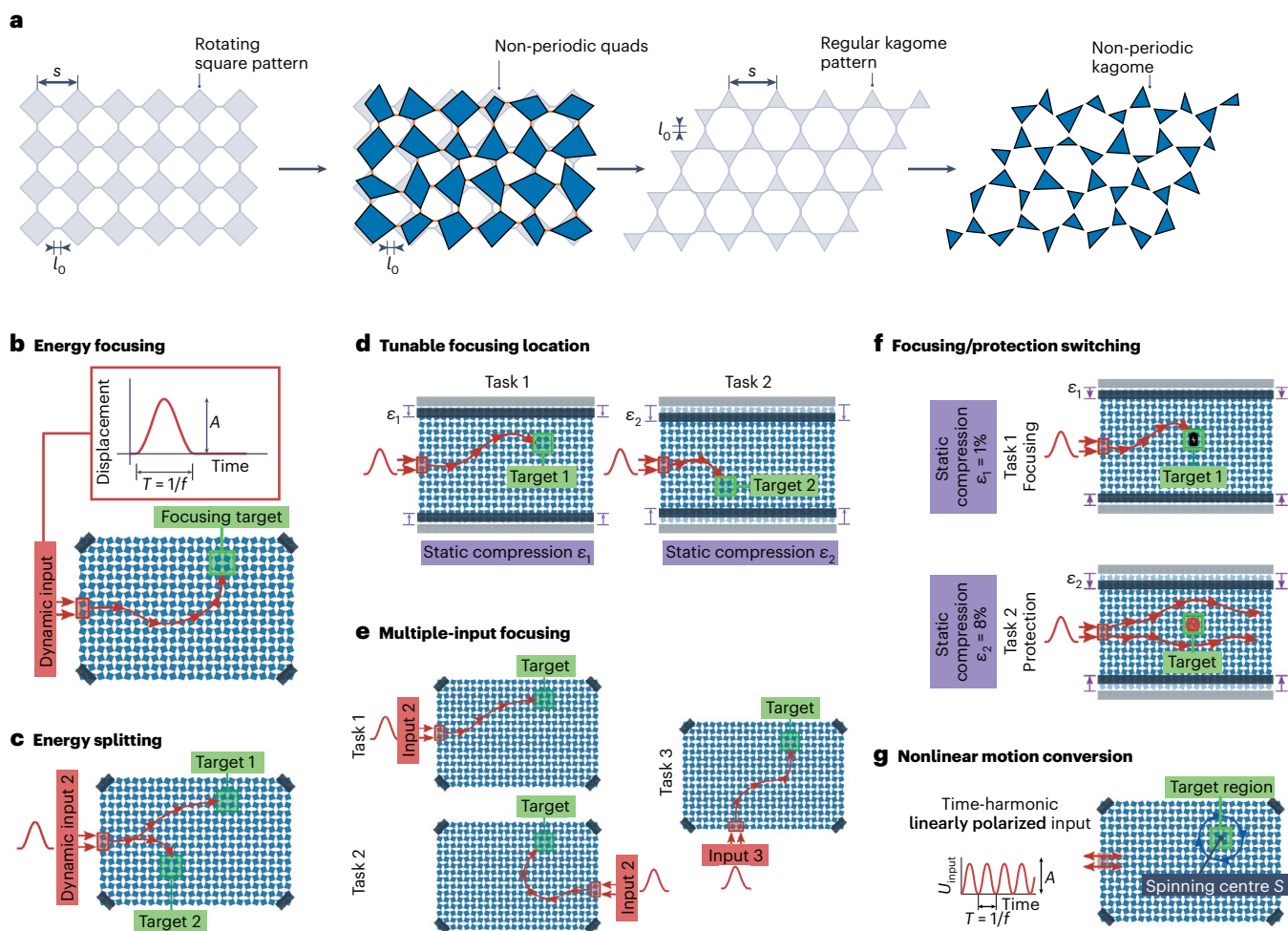


Fig. 1 | Inverse design of reprogrammable nonlinear dynamic metamaterials. **a**, Given a structure consisting of rigid units connected by flexible ligaments, such as a rotating-square pattern or a kagome pattern with spacing s and ligament length l_0 , the proposed framework optimizes the shapes of the rigid units to obtain a metamaterial design with the target nonlinear dynamic responses. **b**, The task of energy focusing, where the goal is to focus the energy provided by a dynamic excitation at a target region. A , amplitude; f , frequency; T , period. **c**, The task of energy splitting, where the goal is to split the energy provided by a dynamic excitation between two target regions. **d**, The task of tunable focusing location, where the goal is to reprogramme the energy-focusing

location with different static pre-compressions. **e**, The task of multiple-input focusing. Here the metamaterial design is optimized to focus the energy of multiple independent boundary excitations at a target region. **f**, The task of focusing/protection switching. Here the metamaterial is optimized to maximize kinetic energy at a target region under a certain static compression and minimize the energy at the same region under a different static compression. **g**, The task of nonlinear motion conversion. Here the metamaterial design is optimized to encode circularly polarized motion at a target region with a linearly polarized large-amplitude harmonic input U_{input} . Figure adapted from ref. 6, Springer Nature Ltd.

framework, such as the effect of the metamaterial topology on the performance of the various nonlinear dynamic tasks, the performance of the tunable focusing location and multiple-input focusing tasks for target regions with different sizes and shapes, and the limitations in performing antagonistic tasks. A natural next step is to apply the inverse design framework to robotics to enhance the performance of soft robots in complex tasks. For instance, the ability to transform simple excitations into some desired motions in specific regions could potentially reduce the need for complex controls in soft robots. The nonlinear dynamic metamaterials obtained by the framework can also be naturally used for energy harvesting and impact mitigation applications.

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Competing interests

The author declares no competing interests.